Comment on "Composite excitation of Josephson phase and spin waves in ferromagnetic Josephson junctions" (S.Hikino, M.Mori, S.Takahashi, and S.Maekawa, arXiv:cond-mat 1009.3551)

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We clarify the applicability of the quasistatic approximation used in Ref. [1], where coupled spin and Josephson plasma waves have been predicted to exist in SIFS Josephson junctions. We show, contrary to the claim of the authors of Ref. [2], that this approximation is very accurate in realistic systems studied experimentally.

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The dynamics of SIFS (or SFIFS) Josephson junctions with ferromagnetic layers has been studied theoretically in Ref. [1]. Here S, I and F stand for a superconducting, thin insulating or ferromagnetic layer, respectively. In particular, it has been shown that weakly damped magneto-plasma oscillations are possible in such a system. That is, oscillations of the magnetic moment M in the F layer and Josephson "plasma" waves turn out to be coupled. The coupled modes (spin and Josephson plasma waves) may result in the peaks on the I-V characteristics of the junction in addition to the Fiske steps.

The same problem has been studied in a recent paper [2]. The authors claim that the electromagnetic (EM) fields in the F layer which excite spin waves in F have been neglected in Ref. [1].

In this Comment, we would like to clarify that:

A) contrary to the statement of Ref. [2], the EM fields in the F film are taken into account in Ref. [1]. Indeed, there could be no coupling between magnetic and plasma modes otherwise.

B) these fields are considered in the quasistatic approximation, which describes the dynamics of realistic junctions rather accurately, while the effects of ac electric fields E, accounted for in Ref. [2], is negligible. The only important ac field is the magnetic induction B which is described by the London equation. The skin effect due to quasiparticle current driven by E can be neglected.

Estimations justifying this approximation are not presented in Ref. [1] for lack of space. Here we give these simple estimations and present a physical explanation of the coupling between the spin and Josephson plasma waves.

The equation for the phase difference φ (Eq.(4) in Ref. [1]) is obtained from the Maxwell equation

$$(\nabla \times \mathbf{B})_z = \frac{4\pi}{c} j_z \tag{1}$$

written in the superconducting regions S (where the magnetic induction $\mathbf{B}$ coincides with the magnetic field $\mathbf{H}$) and from the usual expression for the current through the Josephson junction. The displacement current $j_{dis} = (\epsilon/c)\partial E/\partial t$ is dropped because in metals it is negligible in comparison with the quasiparticle current ($\omega \ll \sigma_Q$), where at $T \precsim \Delta$ the quasiparticle conductance $\sigma_Q \approx \sigma_{Dr}\exp(-\Delta/T)$ with $\sigma_{Dr} = (e^2 n\tau/m) \approx 10^{17}s^{-1}$ for the mean free path $l = v\tau \approx 10^{-6}cm$.

In the quasistatic approximation the expression for $\mathbf{B}(z,t)$ is given by Eq.(3) in Ref. [1]

$$\mathbf{B}_\perp(z,t) = \{\frac{\Phi_0}{4\pi\lambda_L}\mathbf{n}_z\times\nabla_\perp\varphi - \frac{2\pi\tilde{d}_F}{\lambda_L}\mathbf{M}_\perp\}\exp(-\frac{(z-d_F)}{\lambda_L}). \tag{2}$$

It relates the magnetic field in the superconductors and the phase difference $\varphi(t)$. The second term in the curly brackets appears due to the magnetic moment in the F layer(s). Integrating this expression over the square of the superconductors (in the (x,z)-plane perpendicular to the magnetic field) and adding the magnetic moment of the F layer $4\pi\tilde{d}_F L_x M$,, we obtain the usual law of the quantization of the magnetic moment in Josephson junctions: $\Phi \equiv \Phi_S + \Phi_F = \Phi_0 n$, where L_x is the length of the superconductors in the x-direction, Φ_0 is the magnetic flux quantum and n is an integer. Due to the second term in Eq.(2) the Josephson mode is coupled to the spin waves.

Eq.(2) for $\mathbf{B}_\perp$ is obtained by using expression for the current $\mathbf{j}_\perp$ (Eq.(1) in Ref. [1]) which is written in the dirty limit ($\omega\tau \ll 1, kl \ll 1$, where ω, k are characteristic frequency and wave vector, respectively, $l = v\tau$ is the mean free path). The quasiparticle current $\mathbf{j}_{Q\perp} = \sigma_Q(\omega)\mathbf{E}_\perp$ and therefore the transverse electric field $\mathbf{E}_\perp$ ($\nabla \times \mathbf{E}_\perp \neq 0$) is neglected. This approximation is valid if the skin depth δ_{sk} is much larger than the London penetration length λ_L. If the current $\mathbf{j}_{Q\perp}$ is taken into account, then λ_L in Eq.(3) of Ref. [1] should be replaced by $\lambda_\omega = 1/\sqrt{\lambda_L^{-2} + 4\pi i\omega\sigma_Q/c^2}$. The second term is small in comparison with the first one if the frequency $\omega = 2\pi\nu$ is not very high

$$\nu \ll \frac{1}{8\pi^2\sigma_Q}(\frac{c}{\lambda_L})^2 \approx 5\cdot 10^{12}\exp(\frac{\Delta}{T})Hz \tag{3}$$

where we take $\lambda_L \approx 5\cdot 10^{-6}cm$. For the realistic SIFS junctions, where the frequency ν typically is less than one hundred gigahertz [3], this condition is easily fulfilled.

The currents induced in the F layer also change the magnetic induction $\mathbf{B}$. However this change, $\delta\mathbf{B}_F$, is even smaller than the change $\delta\mathbf{B}_{SQ}$ caused by the quasiparticle current in S. Indeed, the change $\delta\mathbf{B}_F$ is determined by the total Meissner current in the F layer $j_{FMeis}d_F$ which is much smaller than $j_{SMeis}\lambda_L$ because $d_F \ll \lambda_L$ (by assumption) and $j_{FMeis} \ll j_{SMeis}$ since the condensate density in F is significantly lower than the density of Cooper pairs in S.

The change $\delta\mathbf{B}_F$ due to skin effect can be neglected if the frequency satisfies the condition

$$\nu \ll \frac{1}{8\pi^2\sigma_F}(\frac{\lambda_L}{d_F})(\frac{c}{\lambda_L})^2 \tag{4}$$

This condition is fulfilled even easier than the condition (2).

Therefore, the currents and ac electric fields in F accounted for in Ref. [2] can be neglected. The only EM field in the F layer, which is essential, is the induction $\mathbf{B}$ determined by Eq.(3) of Ref. [1]. The quasistatic approximation used in deriving this equation is fulfilled for realistic systems (see Ref. [3]) with a great accuracy.

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